A Sensitivity Analysis Method for Equivalent Parameter Extraction of Transient Magnetic Field Problems with Internal Coupled Circuits

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Abstract — A method to extract the equivalent parameters of transient electric circuit and magnetic field coupled problem based on sensitivity computation of the system equations is presented. The field equations are derived by using finite element method. In contrast to traditionally used methods which are based on the physical meanings and physical relationships of the parameters, in this method, the parameters are extracted from the mathematical system equations which include the eddy-current magnetic field equations and internal circuit equations. Its advantage is that it is applicable to complex problems, inside which the reactive energy, iron losses and copper losses are difficult to be isolated and expressed. The extracted parameters can include the effects of eddy-current, internal circuits and mechanical motion. It has been applied to indirect coupling between external circuit and transient magnetic field with internal connected circuits.

I. INTRODUCTION

Electric circuits coupled with magnetic field analysis using finite element method (FEM) have been widely used to simulate power electronic – magnetic – mechanic systems [1]. In such method the electric circuit equations, magnetic field equations and mechanical equations are coupled together to simulate the transient behavior of the system. The circuits can be coupled with the magnetic field directly or indirectly [2-4]. The direct coupling method solves the magnetic field equations and circuit equations simultaneously [2]. In theory this method is efficient and accurate. But if the circuit has highly nonlinear elements, such as power electronic switching components, the nonlinear iterations for the solutions may become very difficult to converge. Indirect coupling between magnetic field and external circuit has the merit of allowing each system to be developed independently and conveniently. Versatile interfaces between the finite element model and external circuit can be established readily with indirect coupling methods [3-4]. The precision of such co-simulation is largely dependent on the accuracy of the parameters extracted from the magnetic field. A precise parameter extraction is a challenge for program developers. Reference [3] presents a parameter extraction method, where the winding inductance of an electric machine is calculated as functions of winding currents and rotor position, which is based on the energy/current perturbation method. In reference [4], the coil inductances are computed as a derivative of the flux linkage with respect to the winding current based on a linearized model. However, in the above methods, due to the influences of a number of factors, such as the eddy current in solid conductors, internally connected circuits, moving objects and permanent magnet (PM) materials, the lumped parameters of the system which have eddy-current magnetic field and internal electric circuits, it is difficult to obtain the parameters of the model precisely. Hitherto, no publications have been found to describe how the parameters of the magnetic field that include all the internal effects, such as eddy current in solid conductors, internal connected circuits, moving objects and permanent magnet (PM) materials, are extracted. Indeed, it is very difficult to deduce the formulation for the computation of such equivalent parameters from the perspective of basic physics.

In this paper a novel methodology to extract the equivalent parameters of the transient magnetic field coupled with internal circuits is presented. The algorithm is based on the mathematical equations of the original FEM and hence there is no need to identify the energy and losses inside the system. By applying sensitivity computation on the system equations of the magnetic field and the internal circuits, the parameters on the coupling ports, which usually are the stranded windings or solid conductors connected to an external circuit, can be directly extracted. Its merit is that the extracted parameters can include the effects of eddy current, internal circuits and mechanical motion within the calculation. Three practical examples are presented to verify the effectiveness of the proposed method.

II. METHODS

A. The Equivalent Circuit When Using Only E.M.F.

At time t^k (k denotes the k^{th} step), the equivalent circuit of the stranded winding or solid conductor of the branch n in the magnetic field region is:

$$u_n^k = R_n i_n^k + e_n^k \quad (1)$$

Assume the current $\boldsymbol{i} = [i_1, i_2, \ldots, i_N]^{\mathrm{T}}$, the branch current of the windings and solid conductors which are connected to the external circuit has an increment $\delta\boldsymbol{i}^k$, then at the same time t^k, taking sensitivity analysis, one has:

$$u_n^k == R_n(i_n^k + \delta i_n^k) + e_n^k + \frac{\partial e_n}{\partial i_1^k}\delta i_1^k + \frac{\partial e_n}{\partial i_2^k}\delta i_2^k + \cdots + \frac{\partial e_n}{\partial i_N^k}\delta i_N^k \quad (2)$$

The increment of the e.m.f. caused by the changes of $\boldsymbol{i}^k$ is:

$$\delta e_n = \frac{\partial e_n}{\partial i_1^k}\delta i_1^k + \frac{\partial e_n}{\partial i_2^k}\delta i_2^k + \cdots + \frac{\partial e_n}{\partial i_N^k}\delta i_N^k \quad (3)$$

B. The Equivalent Circuit When Using Inductances

The equivalent circuit can also be expressed using the lumped circuit parameters:

$$u_n = R_n i_n + e_n = R_n i_n + e_{n(\mathrm{in})} + L_{n1}\frac{\partial i_1}{\partial t} + L_{n2}\frac{\partial i_2}{\partial t} \cdots + L_{nN}\frac{\partial i_N}{\partial t} \quad (4)$$

where the internal e.m.f. $e_{n(\mathrm{in})}$ is attributed to the PMs, current changes in the internal circuits and mechanical movement.

At time t^k, one has, based on the circuit model:

$$u_n^k = R_n i_n^k + e_{n(\mathrm{in})}^k + L_{n1}\frac{i_1^k - i_1^{k-1}}{\Delta t} + L_{n2}\frac{i_2^k - i_2^{k-1}}{\Delta t} \cdots + L_{nN}\frac{i_N^k - i_N^{k-1}}{\Delta t} \quad (5)$$

If the current is $\boldsymbol{i}^k + \delta\boldsymbol{i}^k$, instead of $\boldsymbol{i}^k$, at the same time t^k, then by virtue of sensitivity analysis, one has:

$$u_n^k = R_n(i_n^k + \delta i_n^k) + e_n^k + \delta e_n = R_n(i_n^k + \delta i_n^k) + e_{n(\mathrm{in})}^k$$
$$+ L_{n1}\frac{i_1^k - i_1^{k-1}}{\Delta t} + L_{n2}\frac{i_2^k - i_2^{k-1}}{\Delta t} \cdots + L_{nN}\frac{i_N^k - i_N^{k-1}}{\Delta t}$$
$$+ L_{n1}\frac{1}{\Delta t}\delta i_1^k + L_{n2}\frac{1}{\Delta t}\delta i_2^k \cdots + L_{nN}\frac{1}{\Delta t}\delta i_N^k \qquad (6)$$

Comparing (5) and (6), one obtains the increment of the e.m.f. due to changes of $\boldsymbol{i}^k$ from:

$$\delta e_n = L_{n1}\frac{1}{\Delta t}\delta i_1^k + L_{n2}\frac{1}{\Delta t}\delta i_2^k \cdots + L_{nN}\frac{1}{\Delta t}\delta i_N^k \qquad (7)$$

Comparing (3) and (7), one has:

$$\frac{\partial e_n}{\partial i_m^k} = \frac{L_{nm}}{\Delta t} \qquad i_m = i_1, i_2, \cdots, i_N \qquad \text{and} \qquad L_{nm} = \left(\frac{\partial e_n}{\partial i_m^k}\Delta t\right) \qquad (8)$$

Then, the internal e.m.f. can be calculated by:

$$e_{n(\mathrm{in})}^k = e_n^k - \frac{\sum_{m=1}^{N} L_{nm}^k \left(i^k - i^{k-1}\right)}{\Delta t} \qquad (9)$$

III. EXAMPLES AND RESULTS

A. *Equivalent Inductance of a Three-phase Transformer*

An example to calculate the equivalent inductance of a three-phase transformer is used to validate the program. The transformer is designed with a rated capacity 40 kVA. Its rated voltage in primary windings is 680 V. The rated voltage in secondary windings is 57 V and the power frequency is 50 Hz. The secondary winding is taken as an internal circuit connection in the transformer model. The iron core's conductivity is set to non-zero.

At the beginning, the secondary winding is open-circuit. The connection of the secondary winding is changed to short-circuit at t=0.06 s, as shown in Fig. 1. When the secondary winding is short-circuit, its equivalent inductance value is reduced very drastically and the phase current is increased instantly as shown in Fig. 2, which is consistent with what commonsense predicts.

B. *Coupling Simulation of a Brushless D.C. Motor*

The indirect coupling method using the equivalent parameters extraction method has been applied to simulate a 24-slot, 22-pole PM brushless d.c. machine. The external circuit is shown in Fig. 3. The effect of the free wheeling diodes, which are connected in parallel with the switching elements, is included in the modeling. The simulated magnetic flux distribution is shown in Fig. 4. The calculated inductance waveform is shown in Fig. 5. The simulated phase current waveforms have good agreement with the measured ones, as shown in Fig. 6, which verifies the validity of the proposed approach.

IV. REFERENCES

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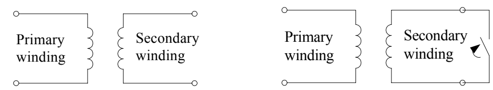
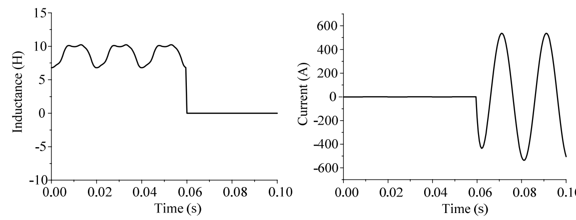


(a) t = 0 (b) t = 0.06 s

Fig. 1. The circuit for one phase of the transformer.



(a) (b)

Fig. 2. Equivalent inductance and phase current of the transformer.(a) Inductance. (b) Phase current.

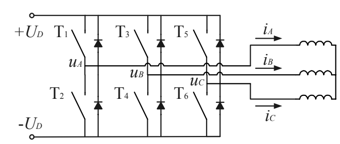


Fig. 3. Representation of the switching elements.

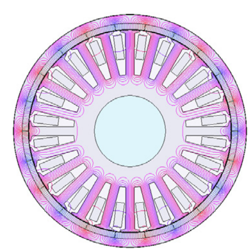


Fig. 4. A brushles d.c. motor and its magnetic flux distribution at full load.

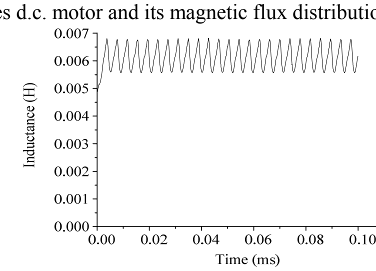
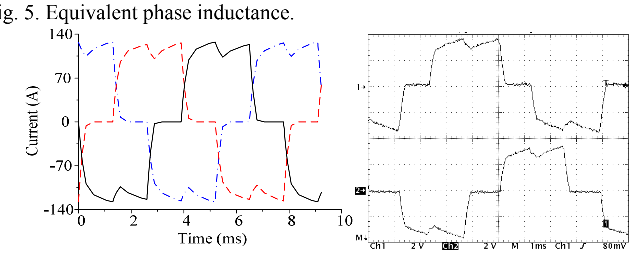


Fig. 5. Equivalent phase inductance.



(a) (b)

Fig. 6. Phase current waveforms.(a) Simulated. (b) Measured (70 A/div, 1 ms/div).